CBOE S&P 500® Implied Correlation Index

Option prices reflect the risk of a stock or stock index. The level of risk conveyed by option prices is often referred to as implied volatility. The implied volatility of a single-stock option simply reflects the market's expectation of the future volatility of that stock's price returns. Similarly, the implied volatility of an index option reflects the market's expectation of the future volatility of that index's price returns. However, index volatility is driven by a combination of two factors: the individual volatilities of index components and the correlation of index component price returns.

Intuitively, one would expect that the implied volatility of an index option would rise with a corresponding change in the implied volatilities of options on the index components. Yet, there are times when index option implied volatility moves and there is no corresponding shift in implied volatilities of options on those components. This outcome is due to the market's changing views on correlation.

The relationship between the implied volatilities of options on an index and the implied volatilities of a weighted portfolio of options on the components of that index, therefore, becomes a measure of the market's expectation of the future correlation of the index components - the "implied" correlation of the index.

The significance of implied correlation is that it reflects changes in the relative premium between index options and single-stock options, providing trading signals for a strategy known as volatility dispersion (correlation) trading. Commonly, a long volatility dispersion trade is characterized by selling at-the-money index option straddles and purchasing at-the-money straddles in options on index components. One interpretation of this strategy is that when implied correlation is high, index option premiums are rich relative to single-stock options. Therefore, it may be profitable to sell the rich index options and buy the relatively inexpensive equity options.

Beginning in July 2009, CBOE will begin disseminating daily values for the CBOE S&P 500® Implied Correlation Index. The CBOE will disseminate two indexes tied to two different maturities – January 2010 **("ICJ")** and January 2011 **("JCJ")**. Both ICJ and JCJ are measures of the expected average correlation of price returns of S&P 500 Index components, implied through SPX option prices and prices of single-stock options on the 50 largest components of the SPX. Each day, CBOE will publish the index values four times per minute, and provide on its website the market value weights of each of the top 50 stocks in the S&P 500 Index. Historical information dating back to 2007 will also be available.

Derivation of the CBOE S&P 500® Implied Correlation Index

An index measures the value of a diversified holding of assets. In the case of a stock index such as the S&P 500, the assets are 500 individual stocks that are among the largest and most actively traded in the world. Generally, the variance of such an index is given by:

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$$\sigma_{Index}^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + 2\sum_{i=1}^{N-1}\sum_{j>i}^{N} w_i w_j \sigma_i \sigma_j \rho_{ij} \quad (2)$$

Where:

σ_i , σ_j = Volatility of ith, jth index components
w_i, w_j = Weight of ith, jth index components
ρ_{ij} = Pair-wise correlation of index components

For the purposes of calculating the CBOE S&P 500 Implied Correlation Index the weight of an index component is determined as follows:

$$w_i = \frac{P_i S_i}{\sum_{i=1}^{50} P_i S_i} \quad (1)$$

where:

P_i = Price of the ith index component
S_i = Float-adjusted shares outstanding of the ith index component

The Index is designed to reflect the market-capitalization weighted average correlation of the Index components, $\rho_{Average}$. As such, equation (2) can be simplified and solved for $\rho_{Average}$:

$$\rho_{Average} = \frac{\sigma_{Index}^2 - \sum_{i=1}^{N} w_i^2 \sigma_i^2}{2\sum_{i=1}^{N-1}\sum_{j>i}^{N} w_i w_j \sigma_i \sigma_j} \quad (3)$$

Sample Calculation

The CBOE S&P 500® Implied Correlation Index measures expected average correlation of the S&P 500 using SPX option implied volatilities and a weighted portfolio of the implied volatilities of options on stocks in an SPX "tracking basket," a subset of the S&P 500 comprised of the 50 largest components as measured by market capitalization.

The options used to calculate ICJ (January 2010) are SPX options expiring in December 2009 and individual stock LEAPS expiring in January 2010. Likewise, JCJ (January 2011) uses SPX options expiring in December 2010 and LEAPS expiring in January 2011. ICJ will be calculated through November 2009 option expiration; JCJ will be calculated through November 2010 option expiration. On the business day immediately following November expiration, CBOE will introduce a new maturity of the CBOE S&P 500 Correlation Index. For example, on Monday, November 23, 2009, CBOE will begin calculating KCJ (January 2012), using SPX options expiring in December 2011 and stock LEAPS expiring in January 2012.

The following sample calculation of ICJ is based on prices at the close of trading on May 29, 2009, and assumes an interest rate of 0.6696%, the yield-to-maturity of the U.S. Treasury

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security maturing closest to January 2010 option expiration. As described below, the stocks comprising the tracking basket would have been selected on April 30, 2009.

Step 1: The 50-stock SPX tracking basket.

On the last business day of each month, the components of the S&P 500 are ranked by market capitalization (closing price times "float-adjusted shares"[1]), with the top 50 stocks comprising the SPX tracking basket for the following month. Additionally, stocks with a market capitalization rank of 51 through 55 are kept in a "replacement pool" in the event that one or more of components is acquired or otherwise removed from the S&P 500 Index. The table in Appendix A lists the SPX tracking basket in effect for May 2009 based on the closing prices of S&P 500 components on April 30, 2009.

Step 2: Select the options to be used in the implied correlation calculation; determine the implied volatility for SPX options, σ_{index}, and the implied volatilities for options on the stocks comprising the SPX tracking basket, σ_i.

For each stock in the SPX tracking basket, the put option with a strike price just below and the call option with a strike price just above the current stock price are selected. The option price is deemed to be the average of the bid / ask quote on CBOE.

Next, for each option, an implied volatility is calculated using the Barone-Adesi Whaley option valuation model. The implied volatilities of each put/call pair are then weighted through a linear interpolation to arrive at a single at-the-money implied volatility for each stock.

For example, on May 29, 2009, the closing price of Apple, Inc. (AAPL) was 135.81. As shown in the following table, the single implied volatility used for AAPL was determined by interpolating the implied volatilities for the 135 Put and 140 Call.

Stock	Expiration	Strike	Put/Call	Mid	Stock price	Implied Vol	Weight	AAPL Implied Vol
AAPL	January 2010	135	P	17.125	135.81	41.73	0.8380	41.49
		140	C	15.8125		40.24	0.1620	

The interpolation weights shown in the above table were determined in the following manner:

$$w_p = \frac{X_c - S}{X_c - X_p} = \frac{140 - 135.81}{140 - 135} = 0.8380$$

$$w_c = 1 - w_p = 1 - 0.8380 = 0.1620$$

where:

X_c = strike price of call option just above the current stock price
X_p = strike price of put option just below the current stock price
S = current stock price

[1] "Float-adjusted" are investable shares as defined by Standard & Poor's; that is, as shares that are freely available to be bought by the investing public. S&P calculates the float-adjusted shares for each component of the S&P 500 components by multiplying a company's shares outstanding by it "investable weight factor," or IWF.

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And…

$$\sigma_{AAPL} = [\ w_p \times \sigma_{AAPL\ 135P}\ ] + [\ w_c \times \sigma_{AAPL\ 140C}\ ] = 41.73 \times 0.8380 + 40.24 \times 0.1620 = \textbf{\textit{41.49}}$$

The SPX options used to calculate ICJ are the put option with a strike price just below and the call option with a strike price just above the forward SPX level to the options' expiration date. The forward index level is determined using at-the-money SPX options prices, where the strike price with the smallest absolute difference between the call and put prices is considered to be the at-the-money strike price. Each option price is deemed to be the average of its bid / ask quote.

Using the Black option valuation model for stock index futures, the implied volatilities for the two SPX options are calculated. Following the same method described for the stock options, a single at-the-money implied volatility for SPX is calculated.

Continuing with the example, the SPX "at-the-money" strike price was 915, and the forward index level for ICJ, F_{ICJ}, was 909.28:

$$F_{ICJ} = X + e^{r*t} * (C - P) = 915 + e^{(0.006696*203/365)} * (72.65 - 78.35) = 909.28$$

where:

X = at-the-money strike price
r = yield to maturity of applicable U.S. Treasury security
t = time to option expiration
C = mid-point of the bid/ask of the at-the-money strike call option
P = mid-point of the bid/ask of the at-the-money strike put option

The same strike-weighted interpolation shown above using equity options is used to determine the at-the-money implied volatility for SPX options. Based on the calculated forward price of 909.28, the implied volatilities of the 900 put and the 915 call were interpolated as shown in the following table to obtain an implied volatility for the SPX of 28.17.

Expiration date	Exercise price	Put/Call	Mid	Forward Index Level	Implied Volatility	Weight	SPX Implied Volatility
19-Dec-09	900	P	71.75	909.28	28.50	0.3814	28.17
19-Dec-09	915	C	72.65		27.96	0.6186	

Step 3: Calculate the capitalization weight, w_i, of each component in the 50-stock basket.

The weight, w_i, assigned to the implied volatility of each component is the float-adjusted market capitalization of that component divided by the total float-adjusted market capitalization of the 50-stock basket:

$$w_i = \frac{P_i S_i}{\sum_{i=1}^{50} P_i S_i}$$

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On May 29, 2009, the total capitalization of the 50-stock basket was $4.15 trillion. The weight of Exxon Mobil Corp (XOM), the largest component in the 50-stock basket, was 8.3% ($343 billion / $4.15 trillion), compared to 4.3% in the S&P 500 Index. It is important to note that the individual stock weights used in the implied correlation calculation are determined relative to the capitalization of the SPX tracking basket, and not the SPX itself.

Step 4: Calculate the implied correlation, ρ_{Average}

Once all of the implied volatilities are determined and component weights are assigned, the actual calculation of the CBOE S&P 500® Implied Correlation Indicator is relatively straightforward.

However, in order to illustrate further, the two terms $\sum_{i=1}^{N} w_i^2\sigma_i^2$ and $2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} w_i w_j \sigma_i \sigma_j$, are expanded below:

$$\sum_{i=1}^{N} w_i^2\sigma_i^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + \ldots + w_{49}^2\sigma_{49}^2 + w_{50}^2\sigma_{50}^2 \quad ,$$

&

$$2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} w_i w_j \sigma_i \sigma_j = 2 \times \left\{ \left[w_1 w_2 \sigma_1 \sigma_2 + w_1 w_3 \sigma_1 \sigma_3 + \ldots + w_1 w_{49} \sigma_1 \sigma_{49} + w_1 w_{50} \sigma_1 \sigma_{50} \right] + \right.$$

$$\ldots + \left[w_2 w_3 \sigma_2 \sigma_3 + \ldots + w_2 w_{49} \sigma_2 \sigma_{49} + w_2 w_{50} \sigma_2 \sigma_{50} \right] +$$

$$\left. \ldots + \left[w_{49} w_{50} \sigma_{49} \sigma_{50} \right] \right\}$$

The table shown in Appendix A includes the implied volatilities and component weights for the 50 SPX components used to calculate the implied correlation index on May 29, 2009. In addition, the implied volatility for SPX, σ_{Index}, was 28.17.

Based on the inputs shown in Appendix A and the values calculated above the Implied Correlation Index for May 29, 2009 was calculated as follows:

$$\rho_{Average} = \frac{\sigma_{Index}^2 - \sum_{i=1}^{N} w_i^2\sigma_i^2}{2\sum_{i=1}^{N-1}\sum_{j>i}^{N} w_i w_j \sigma_i \sigma_j}$$

where:

$\sigma_{Index}^2 = 28.17^2 =$ **793.5489**

$\sum_{i=1}^{N} w_i^2\sigma_i^2 = 30.99^2(0.0827^2) + 35.91^2(0.0385^2) + \ldots + 58.82^2(0.0081^2) + 45.71^2(0.0067^2) =$ **36.93606**

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$$2\sum_{i=1}^{N-1}\sum_{j>i}^{N} w_i w_j \sigma_i \sigma_j = 2*(0.0827*0.0385*30.99*35.91+\ldots+0.0081*0.0067*58.82*45.71) = \mathbf{1272.44500}$$

$$\rho_{Average} = \frac{(793.5489 - 36.93606)}{1272.44500} = 0.\ 594552$$

ICJ = 100*0.594552= 59.46

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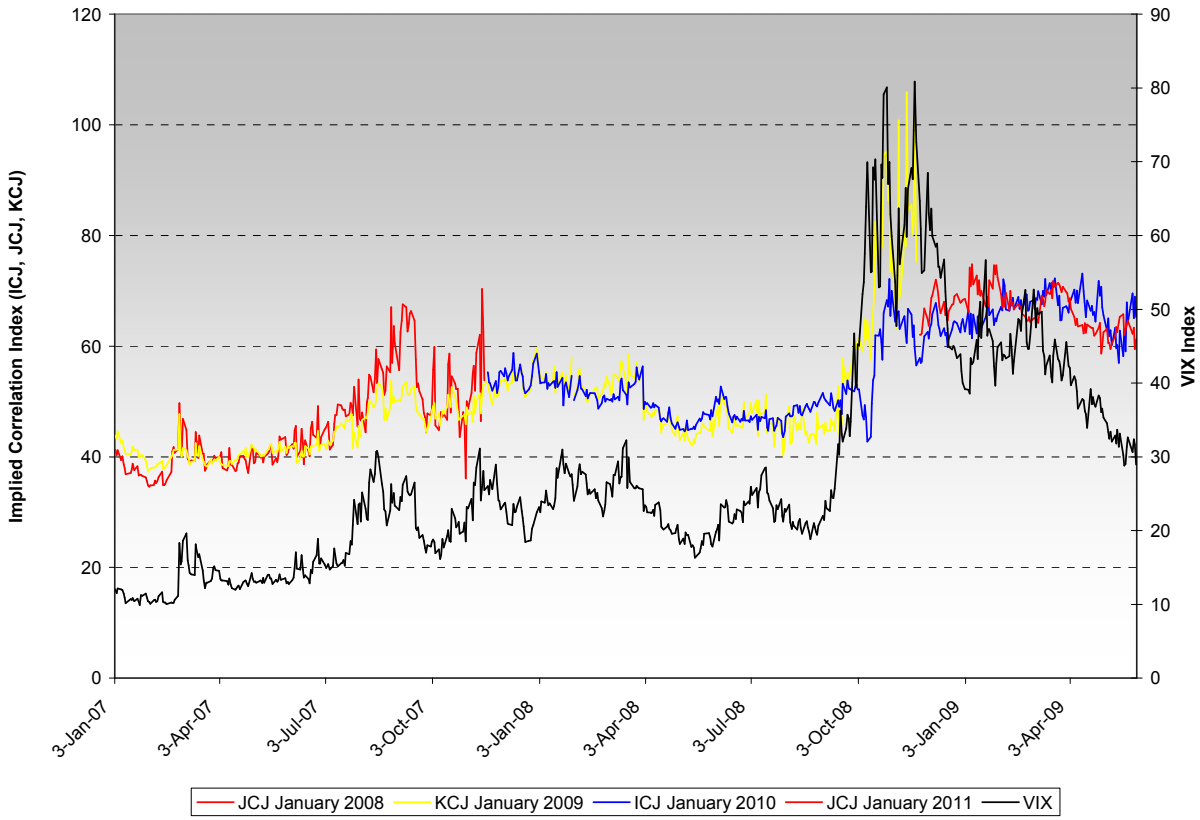
History of the CBOE S&P 500 Implied Correlation Index

CBOE has constructed a history of the CBOE S&P 500 Implied Correlation Index from January 3, 2007 through May 29, 2009. As shown in the chart below, the Implied Correlation Index fluctuated substantially over the last 29 months. The highest closing index level of 105.93 occurred on November 20, 2008 for KCJ with a maturity of January 2009. On this day, the CBOE Volatility Index (VIX) reached its record high close of 80.86, as the S&P 500 closed down 6.71% to 752.44.



Similar to the VIX, implied correlation exhibits a tendency to increase when the S&P 500 decreases. While the inverse relationship to the SPX is similar it is not as strong for the implied correlation indexes. This relationship suggests that the benefits of diversification offered by investing in broad-based equity indexes are limited. The following chart illustrates the inverse relationship between the implied correlation index (ICJ 2010) and the SPX from November 2008 through May 29, 2009.

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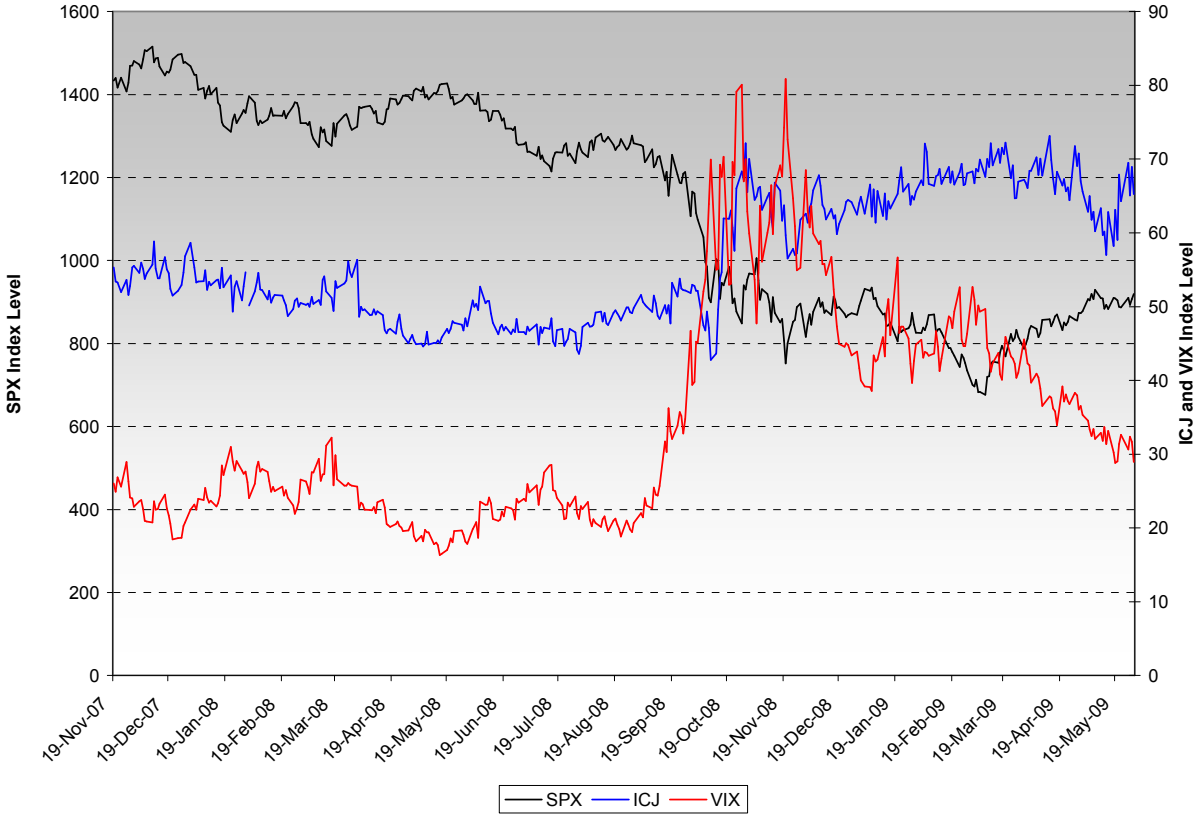

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Appendix A

SPX Stocks used to calculate the CBOE S&P 500 Implied Correlation Index May 29, 2009

Ticker Symbol	Company Name	Share Price	Float-Adjusted Shares Outstanding (millions)	Market Capitalization ($ millions)	Implied Volatility	Basket Weight
AAPL	Apple Inc.	$ 135.81	890.554	$ 120,946	41.49	2.92%
ABT	Abbott Laboratories	$ 45.06	1,545.383	$ 69,635	29.85	1.68%
AMGN	Amgen Inc	$ 49.94	1,033.964	$ 51,636	40.2	1.25%
BAC	Bank of America Corp	$ 11.27	6,401.388	$ 72,144	73.02	1.74%
BMY	Bristol-Myers Squibb	$ 19.92	1,979.509	$ 39,432	33.51	0.95%
CMCSA	Comcast Corp A	$ 13.77	2,880.638	$ 39,666	46.48	0.96%
COP	ConocoPhillips	$ 45.84	1,480.241	$ 67,854	38.91	1.64%
CSCO	Cisco Systems Inc	$ 18.50	5,837.017	$ 107,985	37.87	2.61%
CVS	CVS Caremark Corp.	$ 29.80	1,455.515	$ 43,374	34.25	1.05%
CVX	Chevron Corp	$ 66.67	2,004.559	$ 133,644	32.97	3.22%
DIS	Walt Disney Co	$ 24.22	1,856.335	$ 44,960	40.36	1.08%
GE	General Electric Co	$ 13.48	10,560.425	$ 142,355	48.55	3.43%
GILD	Gilead Sciences Inc	$ 43.10	910.955	$ 39,262	34.96	0.95%
GOOG	Google Inc	$ 417.23	239.623	$ 99,978	32.61	2.41%
GS	Goldman Sachs Group Inc	$ 144.57	502.434	$ 72,637	43.88	1.75%
HD	Home Depot Inc	$ 23.16	1,695.458	$ 39,267	40.12	0.95%
HPQ	Hewlett-Packard Co	$ 34.35	2,396.613	$ 82,324	37.16	1.99%
IBM	Intl Business Machines Corp	$ 106.28	1,341.678	$ 142,594	29.37	3.44%
INTC	Intel Corp	$ 15.72	5,562.000	$ 87,435	37.61	2.11%
JNJ	Johnson & Johnson	$ 55.16	2,765.804	$ 152,562	22.76	3.68%
JPM	JP Morgan Chase & Co	$ 36.90	3,757.923	$ 138,667	52.88	3.35%
KFT	Kraft Foods Inc A	$ 26.11	1,469.388	$ 38,366	27.44	0.93%
KO	Coca-Cola Co	$ 49.16	1,990.606	$ 97,858	24.41	2.36%
LLY	Lilly, Eli & Co	$ 34.57	1,011.134	$ 34,955	32.81	0.84%
LOW	Lowe's Cos Inc	$ 19.01	1,465.681	$ 27,863	45.71	0.67%
MCD	McDonald's Corp	$ 58.99	1,113.622	$ 65,693	26.56	1.58%
MDT	Medtronic Inc	$ 34.35	1,118.225	$ 38,411	35.13	0.93%
MMM	3M Co	$ 57.10	693.792	$ 39,616	31.08	0.96%
MO	Altria Group Inc	$ 17.09	2,066.194	$ 35,311	28.18	0.85%
MON	Monsanto Co.	$ 82.15	547.937	$ 45,013	41.52	1.09%
MRK	Merck & Co Inc	$ 27.58	2,107.712	$ 58,131	35.66	1.40%
MSFT	Microsoft Corp	$ 20.89	7,645.884	$ 159,723	35.91	3.85%
ORCL	Oracle Corp	$ 19.59	3,835.095	$ 75,130	34.62	1.81%
OXY	Occidental Petroleum	$ 67.11	810.295	$ 54,379	45	1.31%
PEP	PepsiCo Inc	$ 52.05	1,556.292	$ 81,005	25.45	1.95%
PFE	Pfizer Inc	$ 15.19	6,745.270	$ 102,461	35.45	2.47%
PG	Procter & Gamble	$ 51.94	2,930.827	$ 152,227	26.92	3.67%
PM	Philip Morris International	$ 42.64	2,003.950	$ 85,448	30.39	2.06%
QCOM	QUALCOMM Inc	$ 43.59	1,649.450	$ 71,900	37.54	1.73%
SGP	Schering-Plough Corp	$ 24.40	1,626.412	$ 39,684	26.95	0.96%
SLB	Schlumberger Ltd	$ 57.23	1,195.990	$ 68,447	46.56	1.65%
T	AT&T Inc	$ 24.79	5,893.307	$ 146,095	33.43	3.52%
UPS	United Parcel Service Inc B	$ 51.14	995.439	$ 50,907	35.45	1.23%
USB	US Bancorp	$ 19.20	1,755.143	$ 33,699	58.82	0.81%
UTX	United Technologies Corp	$ 52.61	942.294	$ 49,574	33.47	1.20%
VZ	Verizon Communications Inc	$ 29.26	2,840.570	$ 83,115	32.12	2.01%
WFC	Wells Fargo & Co	$ 25.50	4,237.777	$ 108,063	63.61	2.61%
WMT	Wal-Mart Stores	$ 49.74	2,235.855	$ 111,211	27.87	2.68%
WYE	Wyeth	$ 44.86	1,331.416	$ 59,727	24.33	1.44%
XOM	Exxon Mobil Corp	$ 69.35	4,941.630	$ 342,702	30.99	8.27%

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